MATH 119: Final

Name: _____

Directions:

* Show your thought process (commonly said as "show your work") when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10
12		10

120

1. Short answer questions:

(a) Suppose you try distributing

$$(x+y)^2 z^2 = (xz^2 + yz^2)^2$$

Why is this incorrect?

(b) Suppose you cancel out the x's to simplify

$$\frac{3+x}{x} = \frac{3+1}{1} = 4$$

Why is this incorrect?

(c) You try simplifying by distributing:

$$[(x-1)^2 + (x+1)]^3 = (x-1)^5 + (x+1)^3 = x^5 - 1^5 + x^3 + 1^3 = x^5 + x^3$$

Circle the two types of mistakes you made and explain why they are mistakes.

2. Evaluate the following:

(a) $\sin\left(\dfrac{\pi}{3}\right)$

(b) $\sec\left(150°\right)$

(c) $\cos^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$

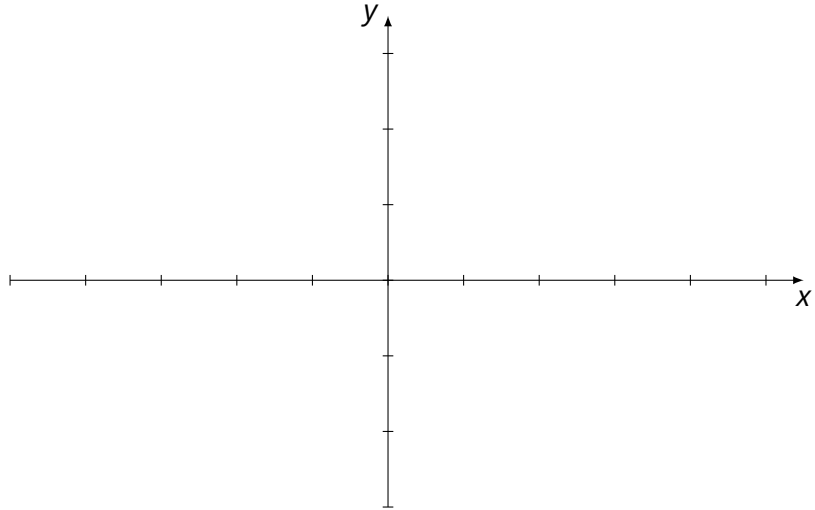(d) $\tan\left(\sin^{-1}\left(\dfrac{\sqrt{2}}{2}\right)\right)$

(e) $\sin\left(\dfrac{5\pi}{2}\right)$

(f) $\sin 75°$

3. Given the function
$$f(x) = 1 + \cos(2x - 2\pi)$$

(a) Graph one period of $f(x)$ using transformations.

(b) What is $f(\pi)$?

4. Suppose $f(x) = x - x^2$

    (a) A person tries to find $f(x + h)$ by writing

$$f(x + h) = x - x^2 + h$$

    This is wrong. What expression (involving $f(x)$) did the person actually write down?

    (b) The person then tries again:

$$f(x + h) = x + h - x + h^2$$

    Explain the reason why this is also incorrect.

    (c) Your turn: Evaluate $f(x + h)$ and fully simplify.

    (d) In general, when you are substituting two or more terms into **(a)** a variable with a power or **(b)** that variable being subtracted, what do you need to not forget?

5. Solve the equation for θ. Check your work if necessary.

(a) $\sqrt{2}\sin\theta + 1 = 0$

(b) $\sin^2\theta = 4 - 2\cos^2\theta$

6. Prove these identities algebraically:

(a) $\dfrac{\sin\theta}{\tan\theta} = \cos\theta$

(b) $\dfrac{\cos x}{\sec x} + \dfrac{\sin x}{\csc x} = 1$

(c) $\cos^4 x - \sin^4 x = \cos 2x$

7. Answer the following:

   (a) A triangle ABC has $\angle A = 90°$, $\angle B = 30°$ and $A = 25$. Solve the triangle and draw a picture of it.

   (b) A sequoia tree casts a shadow 100 feet long. Find the height of the tree if the angle of elevation of the sun is 45°.

   (c) Are $\dfrac{-\pi}{4}$ rad and 315° coterminal? Show with calculations.

8. Answer the following:

    (a) Convert $(\sqrt{8}, \sqrt{8})$ into polar coordinates.

    (b) Convert $r = \dfrac{1}{1 + \sin\theta}$ into rectangular form.

    (c) Convert $r = 6\cos\theta$ into rectangular form.

9. Answer the following:

    (a) Write $1 + i$ in polar form.

    (b) Evaluate $\left(\dfrac{\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}i \right)^{12}$
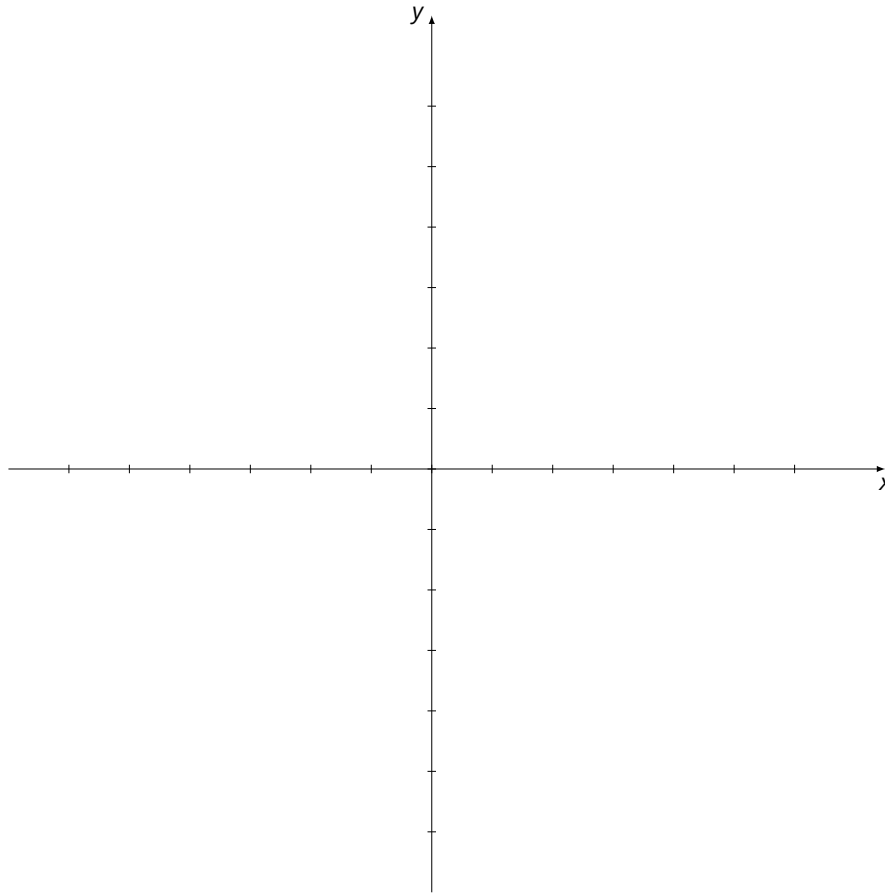
10. Simplify the following trigonometric expressions:

(a) $\sin 20° \cos 40° + \cos 20° \sin 40°$

(b) $\dfrac{\sin(x+y) - \sin(x-y)}{\cos(x+y) + \cos(x-y)}$

11. Here is a pair of parametric equations

$$x = 2t \qquad y = t + 2$$

(a) Sketch the curve represented by the equations.

(b) Find a rectangular coordinate equation for the curve by eliminating the parameter.

12. Answer the following. Do not leave negative exponents.

(a) Simplify

$$\frac{1}{x-1} + \frac{1}{x+1} - \frac{2}{x}$$

(b) Simplify

$$x^2 y \left(\frac{x+1}{y}\right)^{-2} \left(\sqrt{\frac{x}{y}}\right)^4$$

(c) If
$$f(x) = x^2 + 1 \qquad g(x) = 2x^3 \qquad h(x) = 2x - 1 \qquad k(x) = 6x^2$$

fully expand and simplify the following expressions:

i. $f(x)g(x) + h(x)k(x)$

ii. $\dfrac{g(x)f(x) - k(x)h(x)}{[k(x)]^2}$